Charles E. Goodman
Attorney Docket No. BOEI-1-1184
METHODS AND SYSTEMS FOR ANALYZING FLUTTER TEST DATA USING NON-LINEAR TRANSFER FUNCTION FREQUENCY RESPONSE FITTING
Filed: June 24, 2003 -- Serial No. 10/606,067
Sheet 1 of 17
Black Lowe & Graham, PLLC -- (206) 381-3300
"REPLACEMENT SHEET"

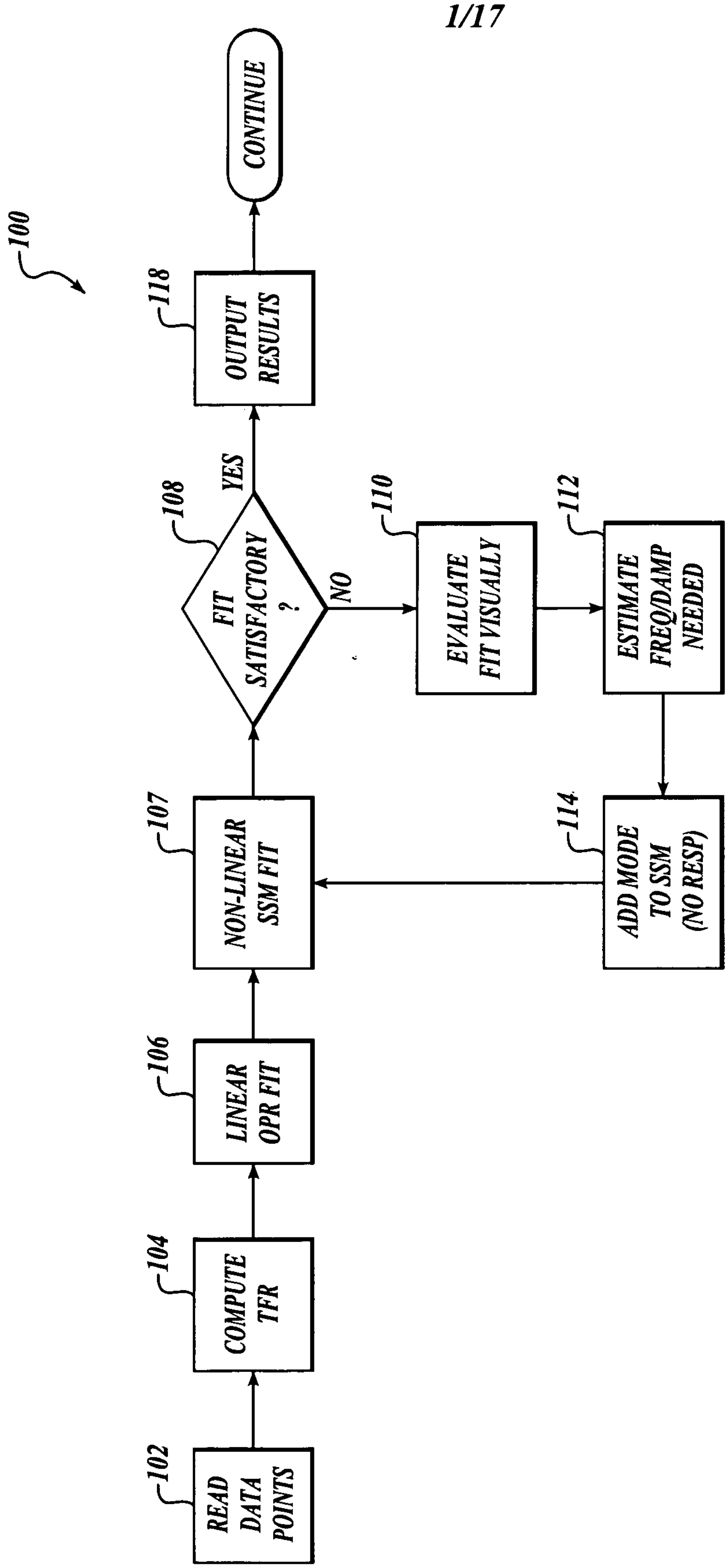


FIG. 1

FIG. 2

FIG. 2A	*FIG. 2B*	*FIG. 2C*	*FIG. 2D*	*FIG. 2E*	*FIG. 2F*	*FIG. 2G*	*FIG. 2H*

$$\frac{\partial \mathrm{Gain}}{\partial x} = \frac{\partial \left(W/N_g \; 20.0 \log_{10}(|Z|) \right)}{\partial x} \qquad (2\text{-}1)$$

$$\frac{\partial \mathrm{Phase}}{\partial x} = \frac{\partial \left(W/N_p \; (180.0/\pi) \tan^{-1}(\mathrm{Im}(Z)/\mathrm{Re}(Z)) \right)}{\partial x} \qquad (2\text{-}2)$$

Where: Gain = gain of transfer function response in dB
Phase = phase of transfer function response in degrees
W = frequency dependent weighting
Ng = gain normalization
Np = phase normalization
Z = complex transfer function frequency response
x = design variable

Since: $|Z| = \sqrt{Z\ Z^*}$

$\log_{10}(u) = \log_{10}(e) \ln(u)$

Gives: $20.0 \log_{10}(|Z|) = 10.0 \log_{10}(e) \ln(Z\ Z^*)$

Then: $$\frac{\partial \mathrm{Gain}}{\partial x} = \frac{\partial \left(W/Ng = 10.0 \log_{10}(e) \;\; \ln(Z\ Z^*) \right)}{\partial x}$$

FIG. 2A

Charles E. Goodman
Attorney Docket No. BOEI-1-1184
METHODS AND SYSTEMS FOR ANALYZING FLUTTER TEST DATA USING NON-LINEAR TRANSFER FUNCTION FREQUENCY RESPONSE FITTING
Filed: June 24, 2003 -- Serial No. 10/606,067
Sheet 4 of 17
Black Lowe & Graham, PLLC -- (206) 381-3300
"REPLACEMENT SHEET"

Since: $$\frac{\partial \ln(u)}{\partial x} = \frac{1.0}{u}\,\frac{\partial u}{\partial x}$$

$$\frac{\partial \tan^{-1}(u)}{\partial x} = \frac{1.0}{1.0+u^2}\,\frac{\partial u}{\partial x}$$

Then: $$\frac{\partial \mathrm{Gain}}{\partial x} = \frac{W\ 10.0\log_{10}(e)}{N_g\ (\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\,\frac{\partial(\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}{\partial x}$$

$$\frac{\partial \mathrm{Phase}}{\partial x} = \frac{W\ (180.0/\pi)\ \mathrm{Re}(Z)^2}{N_p\ (\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\,\frac{\partial(\mathrm{Im}(Z)/\mathrm{Re}(Z))}{\partial x}$$

Since: $$\frac{\partial(u/v)}{\partial x} = \frac{1.0}{v^2}\left(v\,\frac{\partial u}{\partial x} - u\,\frac{\partial v}{\partial x}\right)$$

Gives: $$\frac{\partial \mathrm{Gain}}{\partial x} = \frac{W\ 20.0\log_{10}(e)}{N_p\ (\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\left(\mathrm{Re}(Z)\,\frac{\partial \mathrm{Re}(Z)}{\partial x} + \mathrm{Im}(Z)\,\frac{\partial \mathrm{Im}(Z)}{\partial x}\right)$$

$$\frac{\partial \mathrm{Phase}}{\partial x} = \frac{W\ (180.0/\pi)}{N_p\ (\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2)}\left(\mathrm{Re}(Z)\,\frac{\partial \mathrm{Im}(Z)}{\partial x} + \mathrm{Im}(Z)\,\frac{\partial \mathrm{Re}(Z)}{\partial x}\right)$$

FIG. 2B

There is similarity between the partial of the gain of the response and that of the phase. To uncover the similarity, examine Equation (2-3):

$$\frac{1.0}{z}\frac{\partial z}{\partial x} = \frac{1.0}{Re(Z)+Im(z)j}\left(\frac{\partial Re(Z)}{\partial x} + \frac{\partial Im(Z)}{\partial x}j\right) \tag{2-3}$$

Gives:

$$\frac{1.0}{z}\frac{\partial z}{\partial x} = \frac{1.0}{(Re(Z)^2+Im(Z)^2)}\left(Re(Z)\frac{\partial Re(Z)}{\partial x} + Im(Z)\frac{\partial Im(Z)}{\partial x}\right) + \frac{1.0}{(Re(Z)^2+Im(Z)^2)}\left(Re(Z)\frac{\partial Im(Z)}{\partial x} - Im(Z)\frac{\partial Re(Z)}{\partial x}\right)j$$

Combining the results from Equations (2-1), (2-2) and (2-3) yield Equations (2-4) and (2-5):

$$\frac{\partial Gain}{\partial x} = \frac{W\ 20.0\log_{10}(e)}{N_g}\ Re\left(\frac{1.0}{Z}\frac{\partial Z}{\partial x}\right) \tag{2-4}$$

$$\frac{\partial Phase}{\partial x} = \frac{W\ (180.0/\pi)}{N_p}\ Im\left(\frac{1.0}{Z}\frac{\partial Z}{\partial x}\right) \tag{2-5}$$

FIG. 2C

The complex response of the block diagonal SSM for a specific transfer function is given by Equation (2-6):

$$z_{ij} = \Sigma \left(\frac{N_{ij}{}^{1}}{D^{1}} \right) + d_{ij} \qquad (2\text{-}6)$$

Where:
$$N_{ij}{}^{1} = (c_{i1}{}^{1}\, b_{1j}{}^{1} + c_{i2}{}^{1}\, b_{2j}{}^{1})\, s + (c_{i2}{}^{1}\, b_{1j}{}^{1}\, a_{21}{}^{1} - c_{i1}{}^{1}\, b_{1j}{}^{1}\, a_{22}{}^{1} + c_{i1}{}^{1}\, b_{2j}{}^{1})$$
$$D^{1} = s^{2} - a_{22}{}^{1}\, s - a_{21}{}^{1}$$

For elements in the D matrix the unknown term in Equations (2-4) and (2-5) is given by Equation (2-7) using Equation (2-6):

$$\frac{\partial z_{ij}}{\partial d_{ij}} = 1.0 \qquad (2\text{-}7)$$

For elements in the A, B or C matrices, x1, the unknown term in Equations (2-4) and (2-5) is given by Equation (2-8):

$$\frac{\partial z_{ij}}{\partial x^{1}} = \frac{\partial\, (N_{ij}{}^{1} / D_{1})}{\partial x^{1}} \qquad (2\text{-}8)$$

FIG. 2D

Since: $$\frac{\partial (u/v)}{\partial x} = \frac{1.0}{v^2} \left(v \frac{\partial u}{\partial x} - u \frac{\partial v}{\partial x} \right)$$

Then: $$\frac{\partial z_{ij}}{\partial x^1} = \frac{1.0}{D^1 D^1} \left(D^1 \frac{\partial N_{ij}{}^1}{\partial x^1} - N_{ij}{}^1 \frac{\partial D^1}{\partial x^1} \right)$$

And thus: $$\frac{\partial D^1}{\partial c_{i1}{}^1} = \frac{\partial D^1}{\partial c_{i2}{}^1} = \frac{\partial D^1}{\partial b_{1j}{}^1} = \frac{\partial D^1}{\partial b_{2j}{}^1} = 0.0$$

Simplified: $$\frac{\partial z_{ij}}{\partial x_1} = \frac{1.0}{D^1} \left(\frac{\partial N_{ij}{}^1}{\partial x^1} \right) \quad \text{for } x^1 = c_{i1}{}^1, c_{i2}{}^1, b_{1j}{}^1, b_{2j}{}^1$$

From Equation (2-6) the non-zero partials of the block numerator and denominator are given as Equations (2-9) through (2-16):

$$\frac{\partial N_{ij}{}^1}{\partial c_{i1}{}^1} = b_{1j}{}^1 \, s + (b_{2j}{}^1 - b_{1j}{}^1 \, a_{22}{}^1) \qquad (2\text{-}9)$$

$$\frac{\partial N_{ij}{}^1}{\partial c_{i2}{}^1} = b_{2j}{}^1 \, s + (b_{1j}{}^1 \, a_{21}{}^1) \qquad (2\text{-}10)$$

FIG. 2E

$$\frac{\partial N_{ij}^{1}}{\partial b_{ij}^{1}} = c_{i1}^{1}\, s + (c_{i2}^{1}\, a_{21}^{1} - c_{i1}^{1}\, a_{22}^{1}) \tag{2-11}$$

$$\frac{\partial N_{ij}^{1}}{\partial b_{2j}^{1}} = c_{i2}^{1}\, s + (c_{i1}^{1}) \tag{2-12}$$

$$\frac{\partial N_{ij}^{1}}{\partial a_{21}^{1}} = c_{i2}^{1}\, b_{1j}^{1} \tag{2-13}$$

$$\frac{\partial N_{ij}^{1}}{\partial a_{22}^{1}} = - c_{i1}^{1}\, b_{1j}^{1} \tag{2-14}$$

$$\frac{\partial D^{1}}{\partial a_{21}^{1}} = - 1.0 \tag{2-15}$$

$$\frac{\partial D^{1}}{\partial a_{22}^{1}} = - s \tag{2-16}$$

FIG. 2F

To summarize from Equations (2-4) through (2-16) as Equations (2-17) through (2-30):

$$\frac{\partial Gain_{ij}}{\partial a_{21}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{D^{1}c_{i2}{}^{1}b_{1j}{}^{1}+N_{ij}{}^{1}}{D^{1}D^{1}Z_{ij}}\right) \qquad (2\text{-}17)$$

$$\frac{\partial Phase_{ij}}{\partial a_{21}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{D^{1}c_{i2}{}^{1}b_{1j}{}^{1}+N_{ij}{}^{1}}{D^{1}D^{1}Z_{ij}}\right) \qquad (2\text{-}18)$$

$$\frac{\partial Gain_{ij}}{\partial a_{22}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{-D^{1}c_{i1}{}^{1}b_{1j}{}^{1}+N_{ij}{}^{1}s}{D^{1}D^{1}Z_{ij}}\right) \qquad (2\text{-}19)$$

$$\frac{\partial Phase_{ij}}{\partial a_{22}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{-D^{1}c_{i1}{}^{1}b_{1j}{}^{1}+N_{ij}{}^{1}s}{D^{1}D^{1}Z_{ij}}\right) \qquad (2\text{-}20)$$

$$\frac{\partial Gain_{ij}}{\partial b_{1j}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{c_{i1}{}^{1}s+c_{i2}{}^{1}a_{21}{}^{1}-c_{i1}{}^{1}a_{22}{}^{1}}{D^{1}Z_{ij}}\right) \qquad (2\text{-}21)$$

$$\frac{\partial Phase_{ij}}{\partial b_{1j}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{c_{i1}{}^{1}s+c_{i2}{}^{1}a_{21}{}^{1}-c_{i1}{}^{1}a_{22}{}^{1}}{D^{1}Z_{ij}}\right) \qquad (2\text{-}22)$$

$$\frac{\partial Gain_{ij}}{\partial b_{2j}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{c_{i2}{}^{1}s+c_{i1}{}^{1}}{D^{1}Z_{ij}}\right) \qquad (2\text{-}23)$$

FIG. 2G

$$\frac{\partial Phase_{ij}}{\partial b_{2j}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{c_{i2}{}^{1}s+c_{i1}{}^{1}}{D^{1}z_{ij}}\right) \tag{2-24}$$

$$\frac{\partial Gain_{ij}}{\partial c_{i1}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{b_{1j}{}^{1}s+b_{2j}{}^{1}-b_{1j}{}^{1}a_{22}{}^{1}}{D^{1}z_{ij}}\right) \tag{2-25}$$

$$\frac{\partial Phase_{ij}}{\partial c_{i1}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{b_{1j}{}^{1}s+b_{2j}{}^{1}-b_{1j}{}^{1}a_{22}{}^{1}}{D^{1}z_{ij}}\right) \tag{2-26}$$

$$\frac{\partial Gain_{ij}}{\partial c_{i2}{}^{1}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{b_{2j}{}^{1}s+b_{1j}{}^{1}a_{21}{}^{1}}{D^{1}z_{ij}}\right) \tag{2-27}$$

$$\frac{\partial Phase_{ij}}{\partial c_{i2}{}^{1}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{b_{2j}{}^{1}s+b_{1j}{}^{1}a_{21}{}^{1}}{D^{1}z_{ij}}\right) \tag{2-28}$$

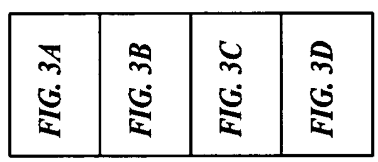
$$\frac{\partial Gain_{ij}}{\partial d_{ij}} = \frac{W\ 20.0\log_{10}(e)}{N_{gij}}\ \mathrm{Re}\left(\frac{1.0}{z_{ij}}\right) \tag{2-29}$$

$$\frac{\partial Phase_{ij}}{\partial d_{ij}} = \frac{W\ (180.0/\pi)}{N_{pij}}\ \mathrm{Im}\left(\frac{1.0}{z_{ij}}\right) \tag{2-30}$$

FIG. 2H

Charles E. Goodman
Attorney Docket No. BOEI-1-1184
METHODS AND SYSTEMS FOR ANALYZING FLUTTER TEST DATA USING NON-LINEAR TRANSFER FUNCTION FREQUENCY RESPONSE FITTING
Filed: June 24, 2003 -- Serial No. 10/606,067
Sheet 11 of 17
Black Lowe & Graham, PLLC -- (206) 381-3300
"REPLACEMENT SHEET"

FIG. 3



The complex response of the PZM is given by Equation (3-1):

$$Z = \frac{N}{D} = \frac{TFG \, \Pi \, N^1}{\Pi \, D^1} \tag{3-1}$$

Where: $N^1 = s^2 + b_1^1 \, s + b_0^1$

$D^1 = s^2 + a_1^1 \, s + a_0^1$

The unknown term in Equations (2-4) and (2-5) is given by Equations (3-2) by using Equation (3-1):

$$\frac{1.0}{Z} \frac{\partial Z}{\partial x} = \frac{1.0}{D \, N} \left(D \frac{\partial N}{\partial x} - N \frac{\partial D}{\partial x} \right) \tag{3-2}$$

The results of Equation (3-2) when the transfer function gain is the design variable, x, is given by the Equation (3-3):

$$\frac{1.0}{z} \frac{\partial Z}{\partial x} = \frac{1.0}{TFG} \quad \text{when } x = TFG \tag{3-3}$$

FIG. 3A

The results of Equation (3-2) when the a numerator block coefficient is the design variable, x, is given by the Equation (3-4):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N}\left(\frac{\partial N}{\partial x}\right) = \frac{1.0}{N^1}\frac{\partial N^1}{\partial x} \quad \text{when } x = b_1{}^1 \text{ or } b_0{}^1 \tag{3-4}$$

Gives: $$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N^1}s \quad \text{when } x = b_1{}^1$$

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{N^1} \quad \text{when } x = b_0{}^1$$

The results of Equation (3-2) when the a denominator block coefficient is the design variable, x, is given by the Equation (3-5):

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = \frac{1.0}{D}\left(\frac{\partial D}{\partial x}\right) = -\frac{1.0}{D^1}\frac{\partial D^1}{\partial x} \quad \text{when } x = a_1{}^1 \text{ or } a_0{}^1 \tag{3-5}$$

Gives: $$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D^1}s \quad \text{when } x = a_1{}^1$$

$$\frac{1.0}{Z}\frac{\partial Z}{\partial x} = -\frac{1.0}{D^1} \quad \text{when } x = a_0{}^1$$

FIG. 3B

To summarize from Equations (2-4), (2-5), (3-3), (3-4) and (3-5) as Equations (3-6) through (3-15):

$$\frac{\partial \mathrm{Gain}}{\partial \mathrm{TFG}} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ \frac{1.0}{\mathrm{TFG}} \tag{3-6}$$

$$\frac{\partial \mathrm{Phase}}{\partial \mathrm{TFG}} = 0.0 \tag{3-7}$$

$$\frac{\partial \mathrm{Gain}}{\partial {b_1}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ \mathrm{Re}\left(\frac{s}{N^1}\right) \tag{3-8}$$

$$\frac{\partial \mathrm{Phase}}{\partial {b_1}^1} = \frac{W\ (180.0/\pi)}{N_p}\ \mathrm{Im}\left(\frac{s}{N^1}\right) \tag{3-9}$$

$$\frac{\partial \mathrm{Gain}}{\partial {b_0}^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\ \mathrm{Re}\left(\frac{1.0}{N^1}\right) \tag{3-10}$$

FIG. 3C

$$\frac{\partial \mathrm{Phase}}{\partial b_0^1} = \frac{W\,(180.0/\pi)}{N_p}\,\mathrm{Im}\left(\frac{1.0}{N^1}\right) \qquad (3\text{-}11)$$

$$\frac{\partial \mathrm{Gain}}{\partial a_1^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\,\mathrm{Re}\left(\frac{-s}{D^1}\right) \qquad (3\text{-}12)$$

$$\frac{\partial \mathrm{Phase}}{\partial a_1^1} = \frac{W\,(180.0/\pi)}{N_p}\,\mathrm{Im}\left(\frac{-s}{D^1}\right) \qquad (3\text{-}13)$$

$$\frac{\partial \mathrm{Gain}}{\partial a_0^1} = \frac{W\ 20.0\ \log_{10}(e)}{N_g}\,\mathrm{Re}\left(\frac{-1.0}{D^1}\right) \qquad (3\text{-}14)$$

$$\frac{\partial \mathrm{Phase}}{\partial a_0^1} = \frac{W\,(180.0/\pi)}{N_p}\,\mathrm{Im}\left(\frac{-1.0}{D^1}\right) \qquad (3\text{-}15)$$

FIG. 3D

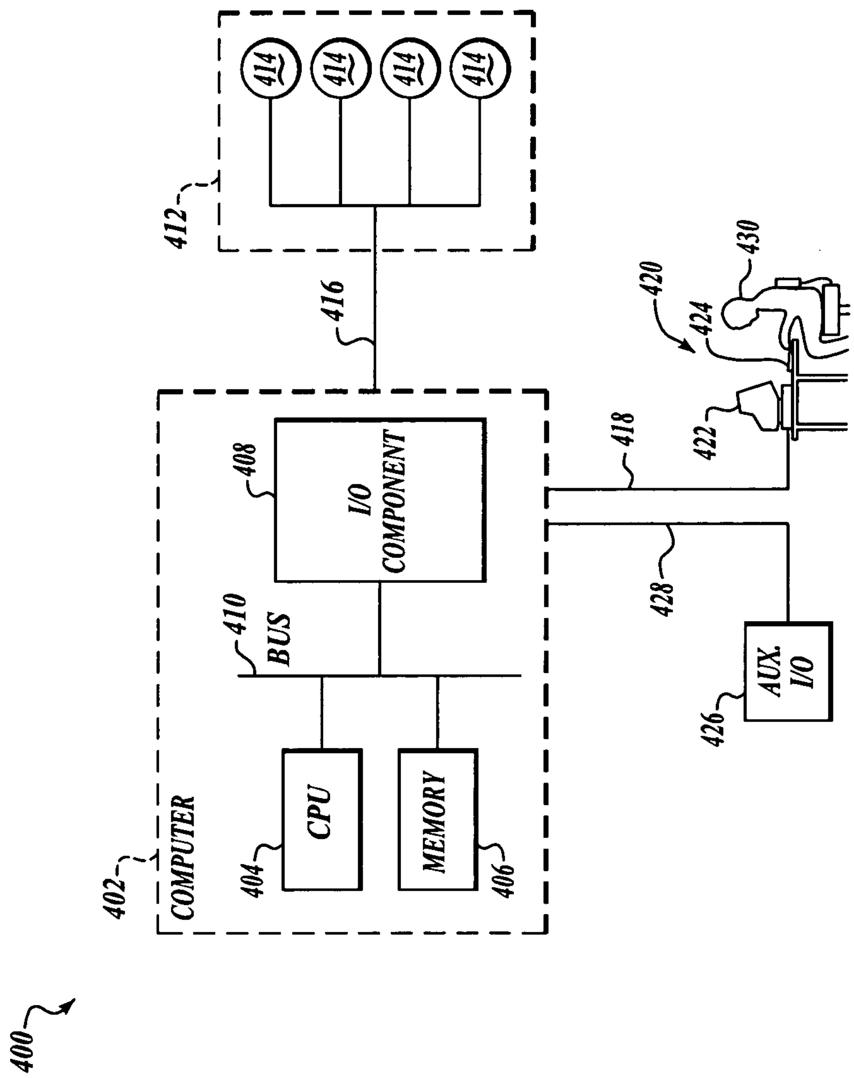

400
402
COMPUTER
404
CPU
406
MEMORY
410
BUS
408
I/O COMPONENT
412
414
414
414
414
416
418
420
422
424
430
426
AUX. I/O
428

FIG. 4

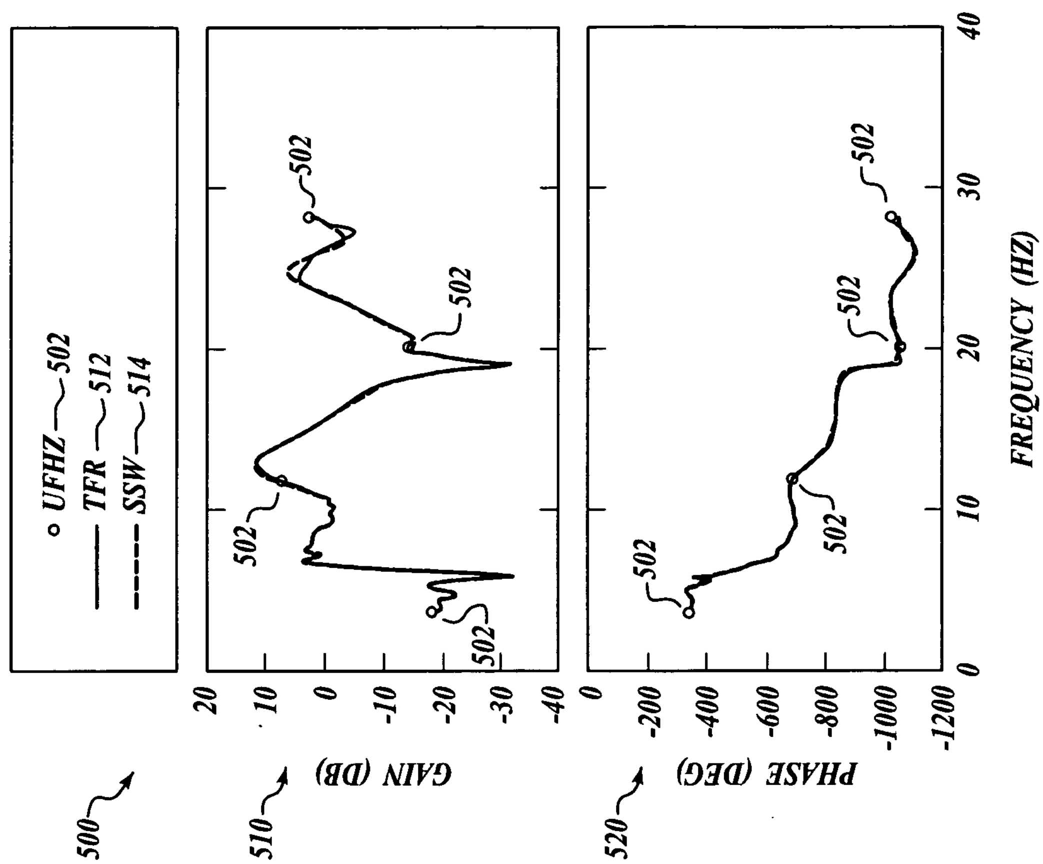

500
510
520
UFHZ 502
TFR 512
SSW 514
GAIN (DB)
PHASE (DEG)
FREQUENCY (HZ)
20
10
0
-10
-20
-30
-40
-200
-400
-600
-800
-1000
-1200
30
40
502

FIG. 5